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A TREATISE ON ALGEBRA.*

This work, as the writers state in their preface, is meant for students who have already an elementary knowledge of algebra and geometry, and as a book of reference for teachers. Among its many merits may be mentioned the completeness of its contents and the exhaustiveness with which each subject is treated. Besides the matter usually treated of in works on algebra, it contains some things not so common, such as infinitesimals and integration, and brings in at the very beginning the idea of function. It also treats of logarithms, continued fractions, etc. in a more extended and helpful manner than usual. The use of checks wherever available and of methods of shortening work, as in detached coefficients, are worthy of note.

The defects of the work as compared with its merits are small. Among them may be mentioned the absence of an index, and the not always clear statement of a proposition or definition. However, the examples accompanying the definitions are very helpful. The work will be of value as a book of reference, but as a text-book it is doubtful whether it can be successfully used, unless preceded by a complete course of elementary collegiate mathematics. F. P. L.

***A Treatise on Algebra.* By Profs. Oliver, Wait, and Jones, of Cornell University. Ithaca, N. Y.: Dudley F. Finch.

SOLUTIONS OF EXERCISES.

ACKNOWLEDGMENTS.

W. W. Beman, 137; G. W. Coakley, 137, 140; H. W. Draughan, 111; W. H. Echols, 144; A. B. Evans, 137; E. Frisby, 137, 140; R. H. Graves, 144; Wm. Hoover, 137, 140, 141; J. N. James, 132, 137; A. C. Lane, 137; F. H. Loud, 143; Artemas Martin, 137; Frank Muller, 39, 144; Chas. Rasinsky, 140; A. D. Risteen, 137; Jas. B. Shaw, Jr., 137, 141; W. O. Whitescarver, 137, 140.

127

A VESSEL of depth a, the top and bottom of which are horizontal planes, is filled with a transparent fluid, the refractive index of which at a depth z below the surface is $1 + z/a$. Two small holes being made in the top, a ray of light enters at one hole, is reflected at the bottom, and emerges at the second hole; show that the distance between the holes must not be greater than

$$2a \log_e (2 + \sqrt{3}). \qquad \text{[\textit{William Hoover.}]}$$

SOLUTION.

Let μ be the declivity of the curved ray at depth z. Then the algebraic expression of the hypothesis is

$$\frac{\cos(\mu + d\mu)}{\cos\mu} = \frac{a + z}{a + z + dz},$$

which is reducible to

$$\tan\mu \, d\mu = \frac{dz}{a + z}.$$

The first integral of this equation is

$$\log \sec \mu = C + \log (a + z),$$

and as the range is greatest when μ vanishes with z

$$C = -\log a,$$

$$\sec \mu = \frac{a + z}{a}.$$

If in this we put for μ its value $\tan^{-1} dz/dx$ and clear of radicals, we get

$$dx = \frac{dz}{\sqrt{[(a + z)^2 - a^2]}};$$

whence

$$x = a \log \frac{(a + z) + \sqrt{[(a + z)^2 + a^2]}}{a},$$

and the greatest range is found by putting $z = a$ to be

$$2x = 2a \log (2 + \sqrt{3}). \qquad \text{[\textit{W. M. Thornton.}]}$$

[The proposer sends the same solution; except that the greatest range is determined by the condition that the ray must not suffer total internal reflection on its return to the surface.]

128

A RHUMB line which cuts the meridians at an angle of 30° is projected upon a plane tangent to the south pole. The centre of projection is: 1° at the centre of the sphere; 2° at the north pole; 3° at an infinite distance on the line of the axis. Required the equation of the projection of the curve in each case.

[*Yale Problems.*]

SOLUTION.

Let $\frac{1}{2}\pi - \theta$ be the latitude of any point on the rhumb line, and φ its longitude, counted from the meridian through the intersection of the curve with the equator. Then, if a be the radius of the sphere, ds an element of the rhumb line, and α its constant angle,

$$\begin{aligned} ds &= a d\theta \sec \alpha \\ &= a \sin \theta \, d\varphi \operatorname{cosec} \alpha. \end{aligned}$$

Eliminating ds,

$$d\varphi = \tan \alpha \frac{d\theta}{\sin \theta};$$

$$\therefore \quad \varphi = \tan \alpha \log \tan \tfrac{1}{2}\theta;$$

and the known equation of the rhumb line is

$$\left.\begin{aligned} r &= a \\ \theta &= 2 \tan^{-1} \varepsilon^{\phi \cot \alpha} \end{aligned}\right\}.$$

Let ρ and φ be the polar co-ordinates in the tangent plane of any projected point; the pole being at the south pole, and φ being equal to the longitude before projection. The equations of the projections are, when $\alpha = 30°$,

1° $$\rho = a \tan \theta = \frac{2a}{\varepsilon^{-\sqrt{3} \cdot \phi} - \varepsilon^{\sqrt{3} \cdot \phi}},$$

2° $$\rho = 2a \tan \tfrac{1}{2}\theta = 2a\varepsilon^{\sqrt{3} \cdot \phi},$$

3° $$\rho = a \sin \theta = \frac{2a}{\varepsilon^{-\sqrt{3} \cdot \phi} + \varepsilon^{\sqrt{3} \cdot \phi}}.$$

[*Frank Muller.*]

144

SHOW that the diurnal path of the shadow of the top of a vertical rod on a horizontal plane will be a rectangular hyperbola, if

$$2 \sin \delta = \sin \left(\frac{\pi}{4} + \varphi\right) + \sin \left(\frac{\pi}{4} - \varphi\right),$$

where φ is the latitude of the place and δ the declination of the sun, assumed to be constant. [*R. H. Graves.*]

SOLUTION.

In the spherical triangle formed by the pole P, the zenith Z, and the position of the sun S, we have $SP = 90° - \delta$, $ZP = 90° - \varphi$.

At sunrise and sunset the end of the shadow of the rod will be at infinity, i. e. $SZ = 90°$.

The asymptotes of a rectangular hyperbola are at right angles, and consequently *radii vectores* to the points at infinity which are parallel to them, also enclose a right angle.

The triangles for sunrise and sunset are similar, and the *radii vectores* being parallel to tangents to SZ at Z, Z must equal 135°. Hence by Napier's rules

$$\begin{aligned}\sin S &= \cos 45^\circ \cos \varphi \\ &= \tfrac{1}{2}[\sin(45^\circ + \varphi) + \sin(45^\circ - \varphi)].\quad Q.\ E.\ D.\end{aligned}$$

[*A. C. Lane.*]

EXERCISES.

151

INTEGRATE the differential

$$\frac{\sin\theta + \cos\theta}{\sin^4\theta + \cos^4\theta}\,d\theta.$$

[*E. Frisby.*]

152

IF r and r' be *radii vectores* in a parabola, and $2f$ the difference of the true anomalies, the area of the parabolic sector between the *radii* is

$$\tfrac{1}{3}\sqrt{(rr')}\sin f\,.\,(r + r' + \sqrt{(rr')}\cos f).$$

[*A. Hall.*]

153

THE normals to a parabola at the points P, Q, R pass through the point O and meet the axis at the points P_1, Q_1, R_1; S is the focus and $4a$ is the length of the *latus rectum;* show that

$$PP_1\,.\,QQ_1\,.\,RR_1 = 8a^2\,.\,SO.$$

[*R. H. Graves.*]

154

FIND the locus of the point from which the four normals drawn to an equilateral hyperbola form an harmonic pencil. [*R. H. Graves.*]

155

A TELLS the truth b times out of a; B, whose veracity is unknown, joins A in affirming the occurrence of an event. What is the probability that the event did occur? [*Artemas Martin.*]

156

A HAWK which is 100 yards vertically above a hare that is running with a uniform velocity in a straight line on a level plane, starts in pursuit, and flying continually toward the hare with a uniform velocity, catches the hare at a point 100 yards distant from where the hare was when the hawk started. What is the length of the curve described by the hawk while in pursuit. [*J. E. Hendricks.*]